preliminaries

probability theory

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probability theory

set theoretic point of view

probability distributions are defined on **sets of elements** that represent **events** *(e.g. the possible outcomes of rolling a die)*

outcomes of an experiment are often coded by using a random variable (statistical/stochastic variable) which is a function from one sample space to another space

events

all possible outcomes of an experiment are given by the sample space Ω and each event A is a subset of Ω.

a probability measure/function/distribution is a function from events to {0,1} interval.

events are sets so combinations of events can be represented by union(∪) or intersection (∩).

propositions

in the Probabilistic Network community, statistical variables are taken to be functions from one sample space to another sample space

For a set of variables V defined on outcomes «true» and «false», possible value assignments are;

for $V = \{V_1,\ldots,V_n\}$, $n \geq 1$

V=true or V=false

$v_i \equiv V_i$=true $\neg v_i \equiv V_i$=false

all compound propositions are constructed from these by applying the binary operators adhering to axioms of propositional logic

binary operators: $\wedge$ (conjunction) $\vee$ (disjunction)

unary operator: $\neg$ (negation)

joint probability distribution

the Probabilistic Network community approaches probability theory from an algebraic point of view by **associating probabilities with logical propositions** instead of sets

restricting discussion to ***binary variables*** *in text for ease of exposition*

joint probability distribution

let V be a set of statistical variables

let x be a logical proposition

let γ be the Boolean algebra of propositions spanned by V

let Pr: $\gamma \rightarrow [0,1]$ be a function such that;

$\Pr(x) \geq 0$, for all $x \in \gamma$, and
Pr(False)=0, Pr(True)=1

$\Pr(x \vee y) = \Pr(x) + \Pr(y)$, for all $x, y \in \gamma$
such that $x \wedge y \equiv$ False

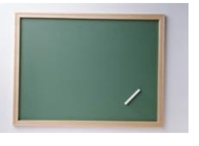
Pr is called a *joint probability distribution* on V for each $x \in \gamma$ the function value $\Pr(x)$ is termed
«the probability of x».

certainty

a probability Pr(*x*) for a logical proposition ***x*** expresses the amount of **certainty** concerning the truth of ***x***

Pr: certainty concerning the truth of an event i.e. the *probability of the truth* of the proposition asserting the occurence of the event

degree of truth is something different than the *probability of truth*.



strictly positive distribution

there are no points of complete certainty in «strictly positive» a.k.a. «non-extreme» probability distributions.

i.e. a strictly positive distribution cannot take values 0 and 1.

i.e. Pr is strictly positive if

$$\Pr(x)=0 \text{ implies } x \equiv \text{False}$$

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conditional probability

let V be a set of statistical variables

let x be a logical proposition

let γ be the Boolean algebra of propositions spanned by V

let Pr be a joint probability distribution on V

for each $x,y \in \gamma$ with $\Pr(y)>0$, the conditional probability of x given y is denoted as $\Pr(x|y)$ and defined as:

$\Pr(x|y) = \Pr(x \wedge y) / \Pr(y)$

i.e. the amount of certainty on the truth of x given that <u>y is known with certainty</u>.

conditional probability

Pr(x|y)=p does <u>**not**</u> mean whenever y is known to be true, the probability of x equals p.

it means; **the probability of *x* equals p if *y* is known and <u>nothing else known may effect the certainty</u> concerning the truth of *x***.

The conditional probability distribution given *y* is denoted as Pr^{y} i.e. it can also be referred as <u>posterior probability distribution</u>.

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independence of propositions

***(mutual)* independence:**

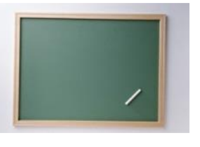
two propositions $x,y\in\gamma$ are called *(mutually)* independent in Pr **if** $\Pr(x \wedge y)=\Pr(x).\Pr(y)$ **else** x and y are called dependent in Pr.

independence of propositions

<u>***(mutual)* independence:**</u>

two propositions $x,y \in \gamma$ are called *(mutually)* <u>independent</u> in Pr **if** $\Pr(x \wedge y) = \Pr(x).\Pr(y)$ **else** x and y are called <u>dependent</u> in Pr.

<u>**conditional independence:**</u>

two propositions $x,y \in \gamma$ are called <u>conditionally independent</u> given the proposition $z \in \gamma$ in Pr **if** $\Pr(x \wedge y|z) = \Pr(x|z).\Pr(y|z)$ **else** x and y are called <u>conditionally dependent</u> given the proposition $z \in \gamma$ in Pr. 

configuration template

A collection of variables $C_{\mathbf{w}}$ is called a configuration template of W set where $W \subseteq V$, and it is shown as $C_{\mathbf{w}} = \wedge_{V_{\mathbf{i}} \in W} V_{\mathbf{i}}$ of all variables in W instead of writing explicity:

$W_{\mathbf{1}}$=value $\wedge$ $W_{\mathbf{2}}$=value $\wedge$ … $\wedge$ $W_{\mathbf{m}}$=value

$C_{\mathbf{w}}$, $W=(W_{\mathbf{1}},\ldots,W_{\mathbf{m}})$ explains all possible assignments to the variables in W, so any configuration $C_{\mathbf{w}}$ of W can be obtained by filling in or updating the necessary values.

For $W=\emptyset$ $C_{\mathbf{w}}$=True which will be used later.

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Let $V=\{V_1,\ldots,V_n\}$ $n\geq 1$ be a set of statistical variables and let Pr be a joint probability distribution on V.

$Pr(C_V)=Pr(V_1 \wedge \ldots \wedge V_n)$
$=Pr(Vn|V_1 \wedge \ldots \wedge V_{n-1}).\ \ldots\ Pr(V_2|V_1).Pr(V_1)$

V_i can be either true(V_i) or false($\neg Vi$)
so there are 2^n equalities.

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independence of variables

let $X,Y,Z \subseteq V$

the set of variables X is *conditionally independent* of the set of variables Y given the set of variables Z **if**

$$\Pr(C_{\mathbf{x}} | C_{\mathbf{y}} \wedge C_{\mathbf{z}}) = \Pr(C_{\mathbf{x}} | C_{\mathbf{z}})$$

i.e. ***once information about Z is available, information about Y is irrelevant w.r.t. X***.

otherwise

X is called *conditionally dependent* on Y given Z.

references

Probabilistic Reasoning

(Probabilistisch Redeneren)

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